Lec 26: 11/25/2013

Cosmic Microwave Background (Cont'd):

So far, we have discussed the effect of acoustic oscillations and Doppler effect on the observed temperature anisotropy of the CMB photons. These provide a qualitative explanation for peaks in the CMB power spectrum. We now discuss additional effects that explain other features in the power spectrum and can help us gain information about fundamental parameters of the "Standard Cosmological Model."

Baryon Loading:

The speed of sound in the baryon-photon fluid is not exactly $\sqrt{\frac{1}{3}}$, as is the case for a photon gas. As it turns out, the energy density in baryons is comparable to that in photons around the time of recombination. This increases the inertia of the fluid, while the pressure remains essentially unchanged:

$$\rho_B = n_B m_B = \eta n_\gamma \overset{(\approx 1\,GeV)}{m_B} = \frac{2\zeta(3)}{\pi^2} T^3 \eta m_B \sim \rho_\gamma \left(= \frac{\pi^2}{15} T^4\right)$$

$$P_B = n_B T = \eta n_\gamma T = \frac{2\zeta(3)}{\pi^2} T^4 \eta << P_\gamma \left(= \frac{\pi^2}{45} T^4\right)$$

To be more quantitative, let us define the following parameter:

$$R = \frac{\rho_B + P_B}{\rho_\gamma + P_\gamma} \simeq \frac{\rho_B}{\rho_\gamma + P_\gamma}$$

By definition, the speed of sound is found from the following expression:

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{\partial P_\gamma}{\partial(\rho_\gamma + \rho_B)}$$

Since $\rho_\gamma, P_\gamma, \rho_B$ are functions of the temperature T, we have:

$$c_s^2 = \frac{\frac{\partial P_\gamma}{\partial T}}{\frac{\partial(\rho_\gamma + \rho_B)}{\partial T}}$$

$$\frac{\partial P_\gamma}{\partial T} = \frac{4\pi^2}{45} T^3$$

$$\frac{\partial(\rho_\gamma + \rho_B)}{\partial T} = \frac{\partial \rho_\gamma}{\partial T} + \frac{\partial \rho_B}{\partial T} = \frac{4\pi^2}{15} T^3 + \frac{6\zeta(3)}{\pi^2} T^2 \eta m_B$$

Thus:

$$c_s^2 = \frac{1}{3} \frac{1}{1+R} \qquad (I)$$

This implies that $c_s^2 < \frac{1}{3}$, which is due to the contribution of bary ons to the energy density of the fluid. The equation for temperature fluctuations now reads (we have neglected damping):

$$\ddot{\left(\frac{\delta T}{T}\right)}_{int} + \frac{k^2}{3(1+R)}\left(\frac{\delta T}{T}\right)_{ints} + \frac{\Phi}{3} = 0 \qquad (\text{II})$$

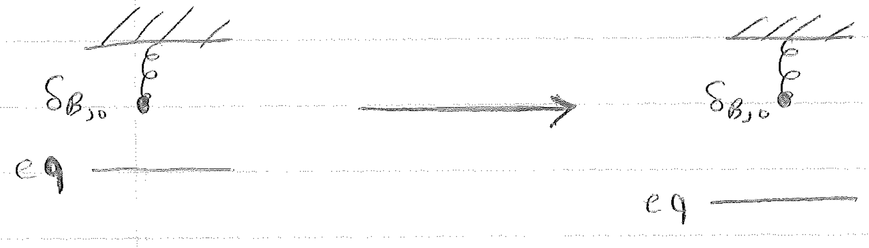
This can be rewritten as:

$$\ddot{\left[\left(\frac{\delta T}{T}\right)_{int} + (1+R)\Phi\right]} + c_s^2 k^2\left[\left(\frac{\delta T}{T}\right)_{int} + (1+R)\Phi\right] = 0$$

Therefore:

$$\left[\left(\frac{\delta T}{T}\right)_{int} + (1+R)\Phi\right]_{(t)} = \left[\left(\frac{\delta T}{T}\right)_{int} + (1+R)\Phi\right]_{(0)} \cos(c_s k t)$$

Using the analogy with harmonic oscillator, the effect of baryons is to lower the equilibrium point of the oscillator in the presence of the external force provided by dark matter perturbations:



$R=0$ $\qquad$ $R\sim 1$

The observed temperature fluctuations then follow:

$$\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_{int} + \Phi = \left(\frac{\delta T}{T}\right)_{obs\,(0)} \cos(c_s k t) + R\Phi\left[\cos(c_s k T) - 1\right]$$

For odd peaks, we have $\cos(c_s k t) = -1$, which results in:

$$\left(\frac{\delta T}{T}\right)_{obs} = -\left(\frac{\delta T}{T}\right)_{obs\,(0)} - 2R\Phi \qquad \text{(odd peaks)}$$

On the other hand, $\cos(c_s k t) = +1$ for even peaks leading to:

$$\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_{obs\,(0)} \qquad \text{(even peaks)}$$

We recall that $\left(\frac{\delta T}{T}\right)_{obs\,(0)} = \frac{1}{3}\Phi$. Therefore:

$$\left|\frac{\left(\frac{\delta T}{T}\right)_{obs,\,odd}}{\left(\frac{\delta T}{T}\right)_{obs,\,even}}\right| = 1 + 6R$$

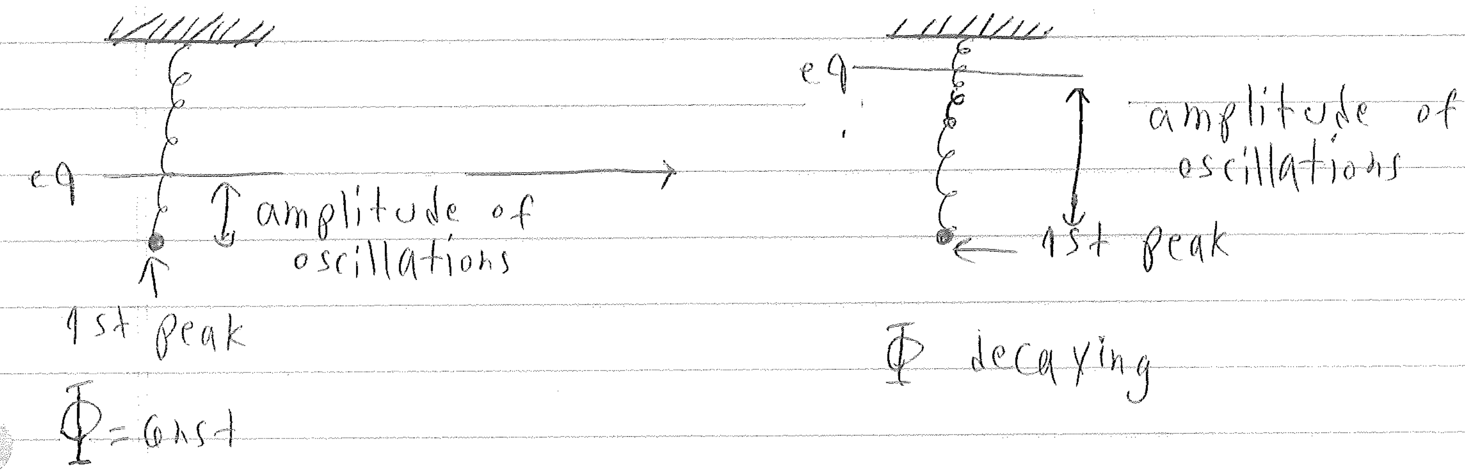
This implies that odd peaks are enhanced relative to even peaks. In reality, R is not a constant as it increases with time. Therefore, the suppression of even peaks relative to odd peaks becomes less pronounced for those modes that start oscillating earlier. We note that increasing Ω_B results in a larger R, and hence more suppression of the 2nd peak (and other even peaks).

Radiation Driving:

Damping due to the expansion of the universe implies that the (n+1)-th peak is typically smaller than the n-th peak. This, however, assumes that oscillations have occured in the matter dominated phase. The higher peaks in the CMB power spectrum, on the other hand, correspond to those modes that started oscillating earlier. The modes that start their oscillations around or before the time of matter-radiation equality t_{eq} behave differently. The reason being that dark matter perturbations do not grow $\propto t^{\frac{2}{3}}$ around or before t_{eq}. As we saw, the growth is only logarithmic when the contribution of radiation to the total energy density is dominant or significant. As a result, the gravitational potential Φ decays before or around t_{eq}:

$$\Phi \propto \frac{\ln t}{t^{\frac{2}{3}}}$$

This has two important consequences. First, going back to the harmonic oscillator analogy, the equilibrium point of the oscillator is shifted upward:



This means that the amplitude of oscillations becomes larger (and keeps increasing because of the continual decrease in Φ). The second effect of Φ decay is that the gravitational redshift becomes less. The two effects combined result in a significant enhancement of $\left(\frac{\delta T}{T}\right)_{obs}$:

$$\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_{int} + \Phi \qquad \left(\frac{\delta T}{T}\right)_{int}\uparrow, \ |\Phi|\downarrow \ \Rightarrow \left(\frac{\delta T}{T}\right)_{obs}\uparrow$$

In fact, the timing of oscillations and decay of Φ is such that the effect on the amplitude of oscillations is maximal hence called "driving". One can show that an increase of ~ 4-5 is expected in the height of the peaks for modes that start their oscillations around t_{eq}.

This enhancement can overcome damping by the Hubble expansion for modes that started to oscillated at $t \sim t_{eq}$. This is provided that a pressureless component like dark matter exists. As a result, enhanced higher peaks (notably the 3rd peak) in the CMB power spectrum are indicative of the dark matter and its abundance.